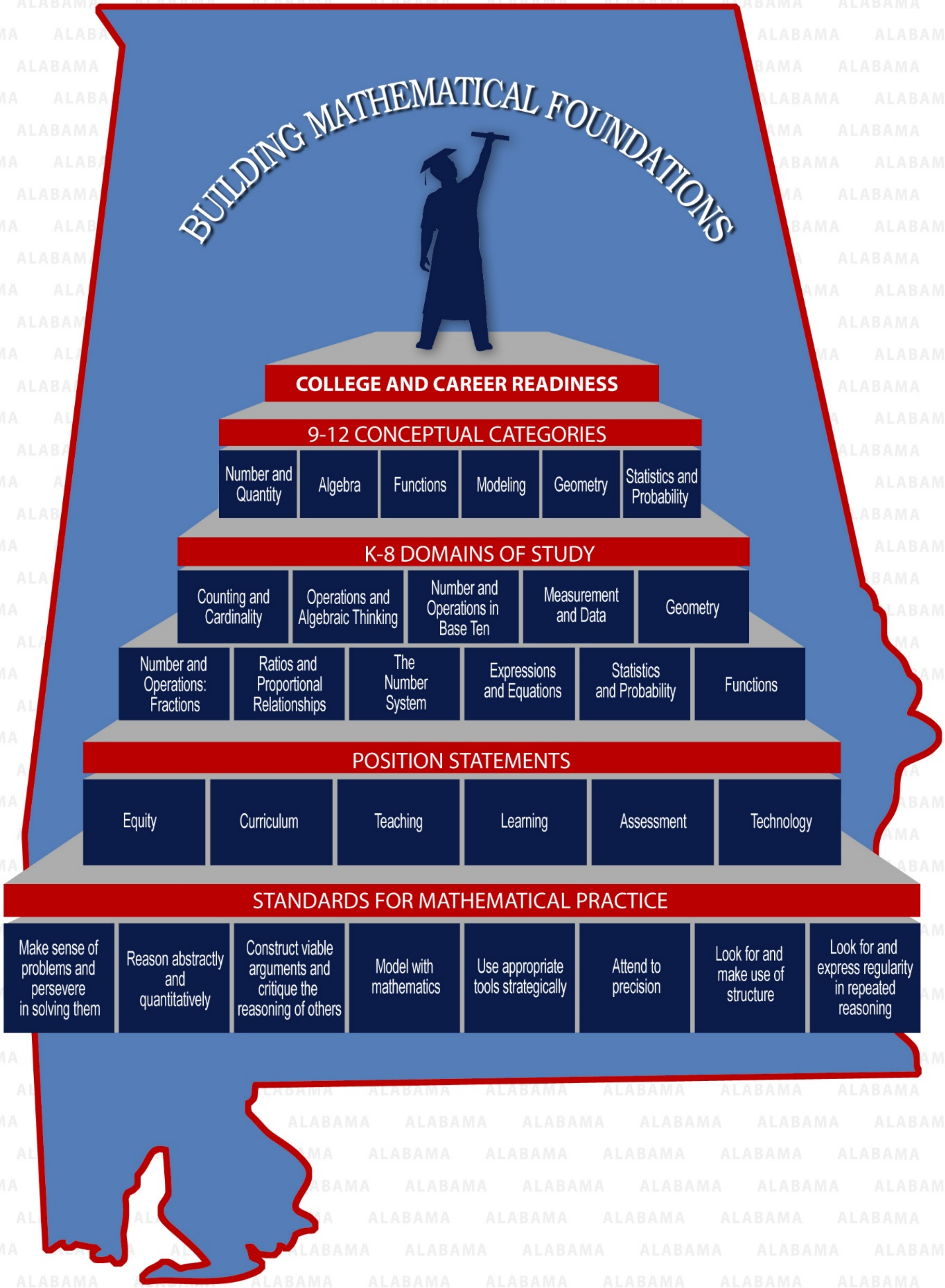
2016 REVISED ALABAMA COURSE OF STUDY
MATHEMATICS



Thomas R. Bice, State Superintendent of Education · Alabama State Department of Education

For information regarding the
Alabama Course of Study: Mathematics
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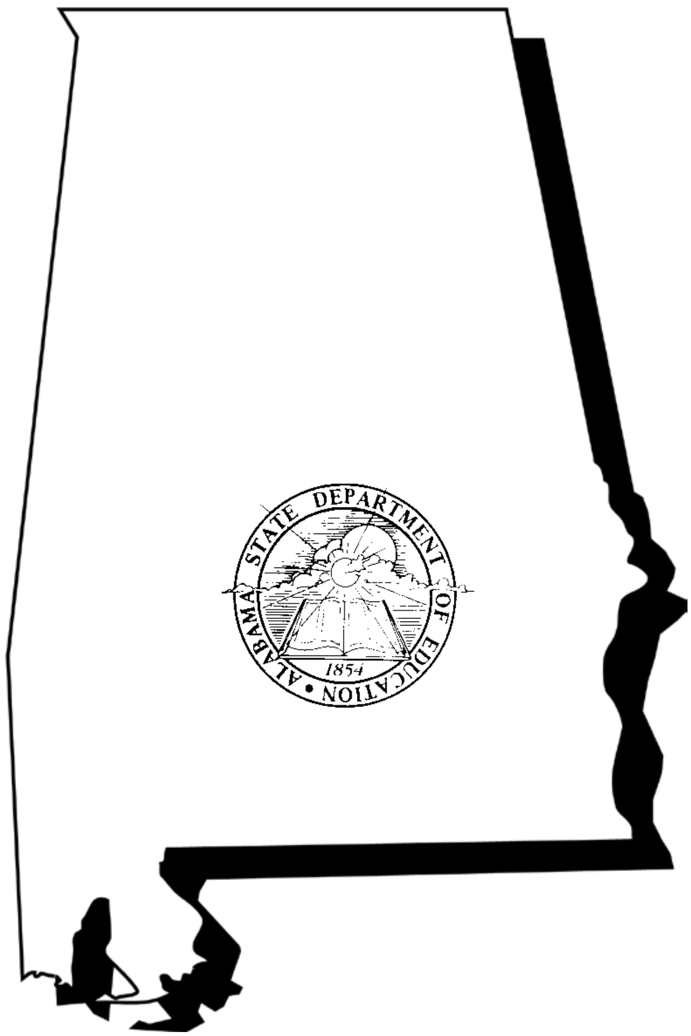
Thomas R. Bice, State Superintendent of Education
Alabama State Department of Education

Alabama Course of Study

Mathematics



Thomas R. Bice
State Superintendent of Education
ALABAMA STATE DEPARTMENT OF EDUCATION

**STATE SUPERINTENDENT
OF EDUCATION'S MESSAGE**

Dear Educator:

The demands of our society and the workplace provide evidence of the need for all Alabama students to achieve the goal of building a strong mathematics foundation. Alabama educators must focus on the teaching of mathematics in ways that ensure students possess adequate preparation to meet future needs and function as problem solvers, decision makers, and lifelong learners. To address this goal, the content of the 2016 *Alabama Course of Study: Mathematics* sets high standards for all students by incorporating *Common Core State Standards for Mathematics (CCSS)* and by addressing mathematical content specific to our state.

The 2016 *Alabama Course of Study: Mathematics*, developed by educators and business and community leaders, provides a foundation upon which quality mathematics programs across the state can be developed. The implementation of the content of this document through appropriate instruction will enable all Alabama students to obtain the mathematical foundations necessary to be college- and career-ready.

Thomas R. Bice
State Superintendent of Education

Alabama Course of Study: Mathematics

Table of Contents

GRADES 9–12 MINIMUM REQUIRED CONTENT

PREFACE

The 2016 *Alabama Course of Study: Mathematics* provides the framework for the Grades K-12 mathematics program in Alabama's public schools. Content standards in this document are minimum and required (*Code of Alabama*, 1975, §16-35-4). They are fundamental and specific, but not exhaustive. In developing local curriculum, school systems may include additional content standards to reflect local philosophies and add implementation guidelines, resources, and activities; which, by design, are not contained in this document.

The 2010 Alabama Mathematics State Standards Task Force made extensive use of the 2010 *Common Core State Standards for Mathematics* document. In addition, the Task Force reviewed the 2009 *Alabama Course of Study: Mathematics* for additional content not specified by the Common Core State Standards, used each member's academic and experiential knowledge, and discussed issues among themselves and with colleagues. Finally, Task Force members compiled what they believe to be the best possible mathematics standards for Alabama's K-12 students. This same group of Alabama professionals, along with additional teachers, returned in 2013, 2015, and in 2016 and revised the standards based on the first years of implementation. This process involved revising standards, moving specific standards to other grades and courses resulting in the 2016 *Alabama Course of Study: Mathematics*.

ACKNOWLEDGMENTS

This document was developed by the 2010 Alabama Mathematics State Standards Task Force composed of 2003 and 2009 Mathematics State Course of Study Committee members and representatives appointed by the Alabama State Board of Education. These members were comprised of early childhood, intermediate school, middle school, high school, and college educators and selected business and industry leaders. The Task Force developed the document during the summer of 2010 and submitted the document to the Alabama State Board of Education for adoption at the November 2010 meeting.

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Charles Creel, Graphic Arts Specialist, Communication Section, assisted in the development of the graphic design.

Susan J. Blankenship, Education Specialist (retired), Alabama Department of Education, edited and proofread the document.

The 2010 *Alabama Course of Study: Mathematics* was reviewed and updated in January 2013, October 2013, January 2015, and again in January 2016.

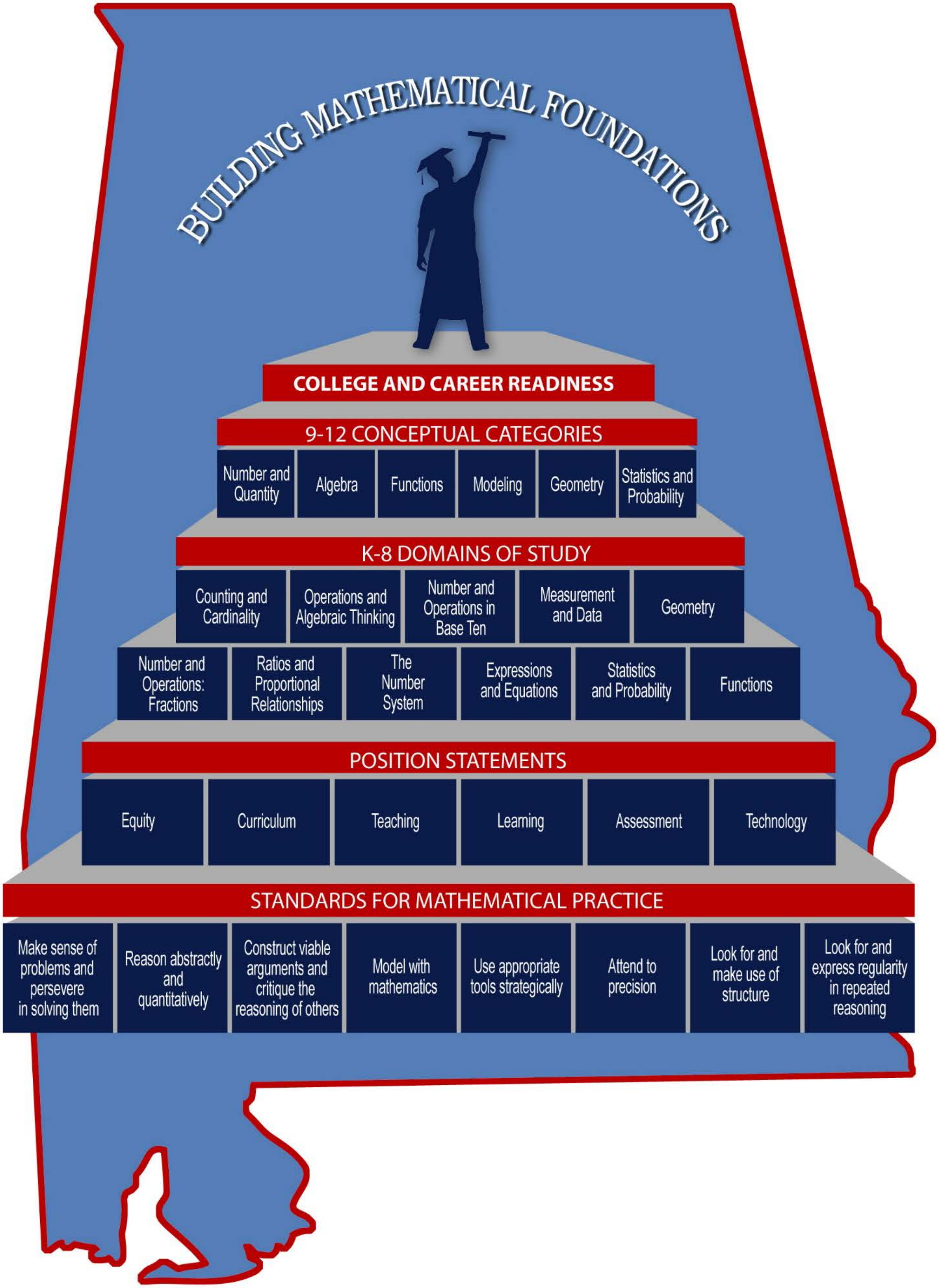
ALABAMA COURSE OF STUDY: MATHEMATICS
GENERAL INTRODUCTION

The 2016 *Alabama Course of Study: Mathematics* defines the knowledge and skills students should know and be able to do upon graduation from high school. Mastery of the standards enables students to succeed in entry-level, credit-bearing academic college courses and in workforce training programs. High school courses addressed in this document include a combination of the *Common Core State Standards for Mathematics* and Alabama specific standards. This additional content is noted in the high school mathematics courses by a map symbol of the state of Alabama ( ), which follows the content standards. All standards contained in this document are:

- Aligned with college and work expectations;
- Written in a clear, understandable, and consistent format;
- Designed to include rigorous content and application of knowledge through high-order skills;
- Formulated upon strengths and lessons of current state standards;
- Informed by high-performing mathematics curricula in other countries to ensure all students are prepared to succeed in our global economy and society; and
- Grounded on sound evidence-based research.

What students can learn at any particular grade level depends upon prior learning. Ideally, each standard in this document might have been phrased in the form, "Students who already know ... should next come to learn ...." However, as research indicates, this approach is unrealistic because all students do not learn in exactly the same way or at exactly the same time in their development. Grade placements for specific topics have been made on the basis of state and international comparisons and on the collective experience and professional judgment of educators, researchers, and mathematicians. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

Mastery of the standards enables students to build a solid foundation of knowledge, skills, and understanding in mathematics. To ensure student success, effective implementation of the 2016 *Alabama Course of Study: Mathematics* requires local education agencies to utilize the minimum required content of this document to develop local curriculum guides.


BUILDING MATHEMATICAL FOUNDATIONS
COLLEGE AND CAREER READINESS
9-12 CONCEPTUAL CATEGORIES
Number and Quantity
Algebra
Functions
Modeling
Geometry
Statistics and Probability
K-8 DOMAINS OF STUDY
Counting and Cardinality
Operations and Algebraic Thinking
Number and Operations in Base Ten
Measurement and Data
Geometry
Number and Operations: Fractions
Ratios and Proportional Relationships
The Number System
Expressions and Equations
Statistics and Probability
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POSITION STATEMENTS
Equity
Curriculum
Teaching
Learning
Assessment
Technology
STANDARDS FOR MATHEMATICAL PRACTICE
Make sense of problems and persevere in solving them
Reason abstractly and quantitatively
Construct viable arguments and critique the reasoning of others
Model with mathematics
Use appropriate tools strategically
Attend to precision
Look for and make use of structure
Look for and express regularity in repeated reasoning

ALABAMA'S K-12 MATHEMATICS CURRICULUM

CONCEPTUAL FRAMEWORK

The goal of Alabama's K-12 mathematics curriculum, "Building Mathematical Foundations," arches across the top of the graphic on the previous page. Student achievement of this goal enhances future opportunities and options for the workplace and for everyday life by enabling all students to be college and career ready. Mathematics content contained in this document is both rigorous and aligned throughout the grades, thus providing students with the necessary steps to acquire the knowledge and skills for developing a strong foundation in mathematics.

The organization of this course of study is based upon the eight CCSS **Standards for Mathematical Practice** and the six **Principles for School Mathematics** found in the National Council of Teachers of Mathematics' (NCTM) document, *Principles and Standards for School Mathematics (PSSM)*. The eight CCSS standards—Make sense of problems and persevere in solving them, Reason abstractly and quantitatively, Construct viable arguments and critique the reasoning of others, Model with mathematics, Use appropriate tools strategically, Attend to precision, Look for and make use of structure, and Look for and express regularity in repeated reasoning—are depicted on the graphic as the foundational blocks of the mathematics program leading to goal achievement. These standards describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. The six NCTM principles—Equity, Curriculum, Teaching, Learning, Assessment, and Technology—build upon the foundation, forming the second level of blocks. They reflect basic tenets fundamental to the design of a quality mathematics program that allows all students the opportunity to reach their mathematical potential. Each of the six principles is further addressed in the Position Statements section found on pages 4-5.

The blocks continue to build upon one another and on the third and fourth levels appear the **K-8 Domains of Study**, which include content describing what students should know and be able to do for each grade or course. The K-8 domains or strands around which groups of related standards are organized are Counting and Cardinality, Operations and Algebraic Thinking, Number and Operations in Base Ten, Measurement and Data, Geometry, Number and Operations: Fractions, Ratios and Proportional Relationships, The Number System, Expressions and Equations, Statistics and Probability, and Functions. The fifth layer of blocks addresses the **9-12 Conceptual Categories**. The 9-12 conceptual categories or strands, also providing organization for groups of related standards, are Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability.

The high school graduate standing on the highest block, **College and Career Readiness**, represents achievement of the goal of developing a strong foundation in mathematics. All students who successfully complete Alabama's K-12 mathematics program are well-equipped for postsecondary mathematics courses as well as prepared for future careers and life situations involving mathematics.

POSITION STATEMENTS

Equity

All Alabama students, with no exception, must have the opportunity to learn relevant and challenging mathematics. In planning for instruction, teachers must set high expectations for students and be mindful of the individual needs, interests, and abilities of all students as they structure their classrooms, plan lessons, design learning activities, and provide reasonable accommodations with the goal of meeting the needs of every student. To this end, content contained within this document is designed to support differentiated instruction that enables and motivates optimum student achievement in the learning of meaningful mathematics.

Curriculum

The 2016 *Alabama Course of Study: Mathematics* is intended to serve as a framework for the development of the mathematics curriculum in local school systems. In addition, teachers also should incorporate into the mathematics curriculum the Literacy Standards found in Appendix C of this document. These standards are designed to supplement students' learning of the mathematical standards by helping them meet the challenges of reading, writing, speaking, listening, and language in the field of mathematics. All content contained in this document is coherent, rigorous, well-articulated across the grades, and focuses on enabling students to make connections between important mathematical ideas. It is essential for educators to select and develop resources that ensure students are capable of making these connections as well as recognizing and applying mathematics concepts in contexts outside the area of mathematics. The mathematics curriculum also must provide students with opportunities to participate in mathematical investigations that lead from knowledge of facts and skills to acquisition of conceptual understanding and problem-solving techniques that enable them to understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

Teaching

The quality and effectiveness of mathematics education in Alabama is influenced by choices made by local school systems and teachers. Effective mathematics teachers develop and maintain their mathematical and pedagogical knowledge, collaborate with colleagues, and seek high-quality professional development opportunities. Effective mathematics teachers use the required content found in this document to plan lessons that engage all students in learning. In elementary grades, effective mathematics teachers recognize the importance of students developing an early interest in and enjoyment of mathematics. In the middle grades and high school, effective mathematics teachers plan relevant classroom activities such as projects and problem-solving situations that require active participation by all students and help them make important connections between mathematics and their personal lives. In addition, effective mathematics teachers consistently reflect on lesson content, lesson activities, and lesson assessments to make necessary adjustments for enhancing student mastery of content.

Learning

Students learn mathematics best when they understand what they are learning. To build upon prior knowledge and experiences, they must be actively engaged in the learning process with meaningful, worthwhile tasks. This engagement affords students opportunities to become confident in their learning and to develop a genuine interest in pursuing ways to solve increasingly difficult problems. Learning mathematics through the use of multiple representations, including algebraic, numerical, graphical, and verbal methods, increases students' abilities to make mathematical connections and become effective communicators of mathematics.

Assessment

Assessment provides teachers and students with information to guide and improve instruction and learning. Effective assessment is planned concurrently with instructional goals. Teachers must plan to use formative assessments during the instructional process and summative assessments at the conclusion of a unit of instruction. Formative assessment is considered a hallmark of effective mathematics instruction and involves the ongoing monitoring of student learning to inform instruction. Information gained from formative assessment is useful to teachers in providing differentiated instruction and to students in the provision of frequent feedback. Varied types of formative assessment should be an integral component of instructional practice in Alabama's mathematics classrooms. Assessment includes, but is not limited to, open-ended problems, constructed-response tasks, selected-response tasks, performance assessments, observations, discussions, journals, and portfolios.

Technology

Technology is an essential component in the teaching and learning of mathematics. Teachers must take advantage of opportunities to heighten student understanding by planning lessons using available technology and making sound instructional decisions about meaningful projects and tasks in which learning is enhanced through the appropriate use of technology. Technology, when used effectively, increases students' understanding of fundamental mathematics concepts, helps them develop an appreciation of mathematics, and inspires them to pursue the study of mathematics for a lifetime.

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices are based on important "processes and proficiencies" that have longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report, *Adding It Up*: *Helping Children Learn Mathematics*. These proficiencies include adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations, and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). The eight Standards for Mathematical Practice are listed below along with a description of behaviors and performances of mathematically proficient students.

Mathematically proficient students:

1. Make sense of problems and persevere in solving them.
These students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. These students consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to obtain the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solve complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships. One is the ability to *decontextualize*, to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents. The second is the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
These students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. These students justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments; distinguish correct logic or reasoning from that which is flawed; and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until the middle or upper grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.
These students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, students might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, students might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas and can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a Web site, and use these to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

These students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. Mathematically proficient students are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. These students also can pause and reflect for an overview and shift perspective. They can observe the complexities of mathematics, such as some algebraic expressions as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

They notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As students work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details and continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The eight Standards for Mathematical Practice described on the previous pages indicate ways in which developing student practitioners of the discipline of mathematics increasingly must engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. It is important that curriculum, assessment, and professional development designers be aware of the need to connect the mathematical practices to the mathematical content standards.

The Standards for Mathematical Content, are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect mathematical practices to mathematical content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, pause for an overview, or deviate from a known procedure to find a shortcut. Thus, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Practice and the Standards for Mathematical Content. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the necessary time, resources, innovative energies, and focus to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

DIRECTIONS FOR INTERPRETING THE MINIMUM REQUIRED CONTENT

The illustrations below and on the next page are intended to serve as guides for interpreting the Grades K-12 minimum required content that all students should learn and be able to do in order to be college- and career-ready. Grades K-8 content standards are grouped according to domain and clusters, while Grades 9-12 standards are grouped by conceptual categories, domains, and clusters.

Domains are large groups of related clusters and content standards. Sometimes standards from different domains may be closely related. In the illustration below, the domain is "Number and Operations in Base Ten."

Clusters are groups of related content standards. Due to the fact that mathematics is a connected subject, standards from different clusters may sometimes be closely related. In the example below, the cluster is "Generalize place value understanding for multi-digit whole numbers."

Content Standards are written beneath each cluster as shown in the following illustrations. Standards define what students should understand (know) and be able to do at the conclusion of a course or grade. Content standards in this document contain minimum required content. The order in which standards are listed within a course or grade is not intended to convey a sequence for instruction. Each content standard completes the phrase "*Students will*."

Standards do not dictate curriculum or teaching methods. For example, one topic may appear before a second in the standards for a given grade, but this does not necessarily mean that the first must be taught before the second. A teacher might prefer to teach the second topic before the first topic, or might choose to highlight connections by teaching both topics at the same time. In addition, a teacher might prefer to teach a topic of his or her own choosing that leads, as a by-product, to students reaching the standards for both topics.

Content Standard Identifiers are found in the brackets following each content standard. In the illustration below for Grade 4, this information identifies the student grade level and the CCSS mathematics domain and the standard number. For example, the first content standard in the example is followed by content standard identifier [4-NBT1] to indicate the student grade level as fourth (4), the CCSS domain as Number and Operations in Base Ten (NBT), and the CCSS number as one (1).

GRADE 4

Students will:

Domain

Cluster

Content Standards

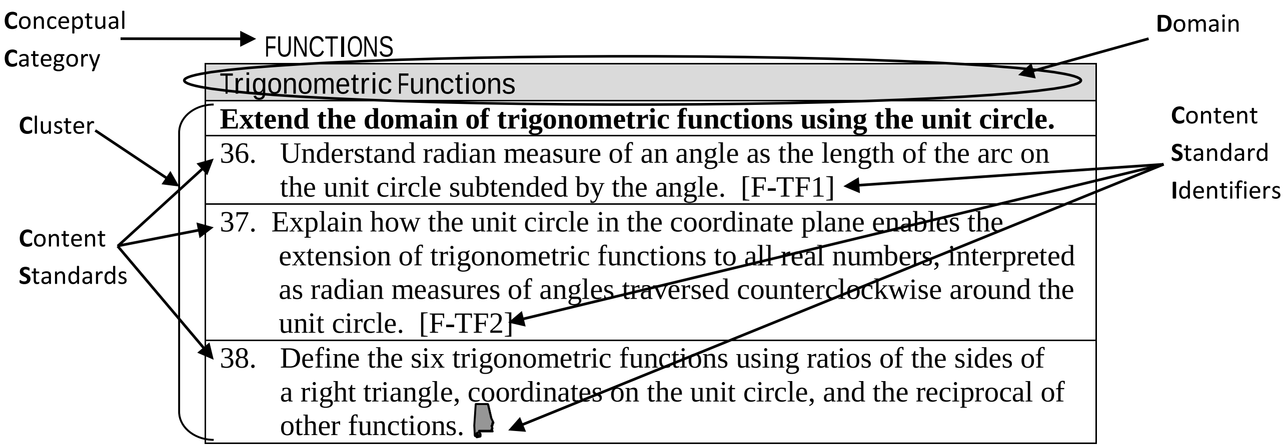
Content Standard Identifiers

Number and Operations in Base Ten
Generalize place value understanding for multi-digit whole numbers.
6. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. [4-NBT1]
7. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meaning of the digits in each place using >, =, and < symbols to record the results of comparisons. [4NBT-2]
8. Use place value understanding to round multi-digit whole numbers to any place. [4-NBT3]

For high school courses, as in the illustration below, the bracketed information identifies the conceptual category by which the standard is grouped, the CCSS domain, and the CCSS number. Conceptual categories are described in the high school mathematics section of this document on pages 67-80. In the illustration below for Algebra II With Trigonometry, the second content standard is followed by content standard identifier [F-TF2] to indicate the CCSS conceptual category as Functions (F), the domain as Trigonometric Functions (TF), and the CCSS number as two (2). Required content added from the 2009 *Alabama Course of Study: Mathematics* is noted at the end of a standard by a state of Alabama symbol (), as shown in the third content standard of the example.

ALGEBRA II WITH TRIGONOMETRY

Students will:



STANDARDS FOR HIGH SCHOOL MATHEMATICS

Categories of Standards for High School Mathematics

The high school mathematics standards are grouped according to six conceptual categories. These categories provide a coherent view of high school mathematics content. A student's work with functions, for example, crosses a number of traditional course boundaries, potentially up to and including Precalculus. The conceptual categories, as listed below, are described in detail on the following pages.

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Additional Standards for High School Mathematics

High school content standards specify the mathematics that all students should learn and be able to do in order to be college and career ready. Additional mathematics content that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by a plus symbol (+), as in this example:

Example: (+) Represent complex numbers on the complex plane in rectangular and polar form, including real and imaginary numbers.

All standards without a plus symbol (+) are included in the mathematics curriculum for all college- and career-ready students. Some standards with a plus symbol (+) also appear in courses intended for all students.

Modeling Standards for High School Mathematics

The Standards for Mathematical Practice include a standard that requires the modeling of mathematics. Detailed information regarding modeling is located on pages 74-75 of this document. Modeling is best interpreted, not as a collection of isolated topics, but rather having relevance to other standards. Specific modeling standards appear throughout the high school mathematics standards, and they are indicated by an asterisk (*). The asterisk (*) may appear after a particular standard as shown in the example below, or it may appear on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Example: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

CONCEPTUAL CATEGORY: NUMBER AND QUANTITY

Numbers and Number Systems

During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number," as in 1, 2, 3…. Soon after that, 0 is used to represent "none," and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them by means of their decimal representations with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways. They have the commutative, associative, and distributive properties; and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems (CAS) provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments; to help understand the workings of matrix, vector, and complex number algebra; and to experiment with non-integer exponents.

Quantities

In real-world mathematics problems, the answers are usually not numbers, but quantities or numbers with units involving measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, including acceleration, currency conversions, and derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they recognize a need or ascertain a problem, and then must both pose and find a solution for the situation or problem encountered. For example, to find a good measure of overall highway safety, students might propose measures to collect data regarding fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for private companies and governmental agencies, which must conceptualize relevant needs and create or choose suitable measures for them.

NUMBER AND QUANTITY OVERVIEW

The Number and Quantity conceptual category focuses on the four domains listed below in the column to the left. The Number and Quantity domains listed in bold print on the shaded bars are The Real Number System, Quantities, The Complex Number System, and Vector and Matrix Quantities. Immediately following the domain and enclosed in brackets is an abbreviation denoting the domain. Identified below each domain are the clusters that serve to group related content standards.

The Standards for Mathematical Practice are listed below in the column to the right. These mathematical practice standards should be incorporated into classroom instruction of the content standards.

Content Standard Domains and Clusters

The Real Number System [N-RN]

- Extend the properties of exponents to rational exponents.
- Use properties of rational and irrational numbers.

Quantities [N-Q]

- Reason quantitatively and use units to solve problems.

The Complex Number System [N-CN]

- Perform arithmetic operations with complex numbers.
- Represent complex numbers and their operations on the complex plane.
- Use complex numbers in polynomial identities and equations.

Vector and Matrix Quantities [N-VM]

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

CONCEPTUAL CATEGORY: ALGEBRA

Expressions

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions. These resources are valuable aids to help in performing complicated, algebraic manipulations and understanding how algebraic manipulations behave.

Equations and Inequalities

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

ALGEBRA OVERVIEW

The Algebra conceptual category focuses on the four domains listed below in the column to the left. The Algebra domains listed in bold print on the shaded bars are Seeing Structure in Expressions, Arithmetic With Polynomials and Rational Expressions, Creating Equations, and Reasoning With Equations and Inequalities. Immediately following the domain and enclosed in brackets is an abbreviation denoting the domain. Identified below each domain are the clusters that serve to group related content standards.

The Standards for Mathematical Practice are listed below in the column to the right. These mathematical practice standards should be incorporated into classroom instruction of the content standards.

Content Standard Domains and Clusters

Seeing Structure in Expressions [A-SSE]
- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

Arithmetic With Polynomials and Rational Expressions [A-APR]
- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.

Creating Equations [A-CED]
- Create equations that describe numbers or relationships.

Reasoning With Equations and Inequalities [A-REI]
- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.
- Solve systems of equations.
- Represent and solve equations and inequalities graphically.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

CONCEPTUAL CATEGORY: FUNCTIONS

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, for example by a graph indicating the trace of a seismograph; by a verbal rule as in, "I'll give you a state, you give me the capital city;" by an algebraic expression such as $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system (CAS) can be used to experiment with properties of these functions and their graphs. These technologies are valuable aids to help in the building of computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

FUNCTIONS OVERVIEW

The Functions conceptual category focuses on the four domains listed below in the column to the left. The Functions domains listed in bold print on the shaded bars are Interpreting Functions; Building Functions; Linear, Quadratic, and Exponential Models; and Trigonometric Functions. Immediately following the domain and enclosed in brackets is an abbreviation denoting the domain. Identified below each domain are the clusters that serve to group related content standards.

The Standards for Mathematical Practice are listed below in the column to the right. These mathematical practice standards should be incorporated into classroom instruction of the content standards.

Content Standard Domains and Clusters

Interpreting Functions [F-IF]

- Understand the concept of a function and use function notation.
- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.

Building Functions [F-BF]

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Linear, Quadratic, and Exponential Models [F-LE]

- Construct and compare linear, quadratic, and exponential models and solve problems.
- Interpret expressions for functions in terms of the situation they model.

Trigonometric Functions [F-TF]

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

CONCEPTUAL CATEGORIES: MODELING

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

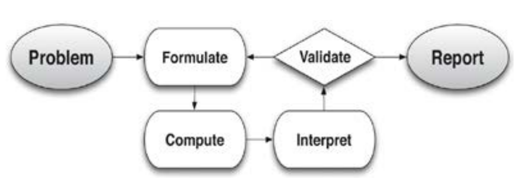
A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even simple models involve making choices such as the decision to model a coin as a three-dimensional cylinder or as a two-dimensional disk. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Examples of such situations might include the following:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against every other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis such as turnaround space required for an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors, including “How precise an answer do we want or need?” “What aspects of the situation do we most need to understand, control, or optimize?” “What resources of time and tools do we have?” The range of models that can be created and analyzed is also constrained by the limitations of our mathematical, statistical, and technical skills as well as the ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.



The basic modeling cycle is summarized in the diagram to the right. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies until cut-off mechanisms, such as pollution or starvation intervene, follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

A variety of powerful tools can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena. Such tools include graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software.

Modeling Standards. *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by an asterisk* (*).

CONCEPTUAL CATEGORY: GEOMETRY

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts. These include interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate that through a point not on a given line there is exactly one parallel line. Spherical geometry, in contrast, has no parallel lines.

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles, and therefore shapes generally. Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria, angle-side-angle (ASA), side-angle-side (SAS), and side-side-side (SSS), are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to nonright triangles by the Law of Cosines. Together, the Law of Sines and the Law of Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that side-side-angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools. These tools allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations

The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

GEOMETRY OVERVIEW

The Geometry conceptual category focuses on the six domains listed below in the column to the left. The Geometry domains listed in bold print on the shaded bars are Congruence; Similarity, Right Triangles, and Trigonometry; Circles; Expressing Geometric Properties With Equations; Geometric Measurement and Dimension; and Modeling With Geometry. Immediately following the domain and enclosed in brackets is an abbreviation denoting the domain. Identified below each domain are the clusters that serve to group related content standards.

The Standards for Mathematical Practice are listed below in the column to the right. These mathematical practice standards should be incorporated into classroom instruction of the content standards.

Content Standard Domains and Clusters

Congruence [G-CO]
- Experiment with transformations in the plane.
- Understand congruence in terms of rigid motions.
- Prove geometric theorems.
- Make geometric constructions.

Similarity, Right Triangles, and Trigonometry [G-SRT]
- Understand similarity in terms of similarity transformations.
- Prove theorems involving similarity.
- Define trigonometric ratios and solve problems involving right triangles.
- Apply trigonometry to general triangles.

Circles [G-C]
- Understand and apply theorems about circles.
- Find arc lengths and areas of sectors of circles.

Expressing Geometric Properties With Equations [G-GPE]
- Translate between the geometric description and the equation for a conic section.
- Use coordinates to prove simple geometric theorems algebraically.

Geometric Measurement and Dimension [G-GPE]
- Explain volume formulas and use them to solve problems.
- Visualize relationships between two-dimensional and three-dimensional objects.

Modeling With Geometry [G-MG]
- Apply geometric concepts in modeling situations.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

CONCEPTUAL CATEGORY: STATISTICS AND PROBABILITY*

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell-shaped; and it might be summarized by a statistic measuring center such as mean or median and a statistic measuring spread such as standard deviation or interquartile range. Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data. In critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed, as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability. The use of technology makes it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling

Functions may be used to describe data. If the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

STATISTICS AND PROBABILITY OVERVIEW

The Statistics and Probability conceptual category focuses on the four domains listed below in the column to the left. The Statistics and Probability domains listed in bold print on the shaded bars are Interpreting Categorical and Quantitative Data, Making Inferences and Justifying Conclusions, Conditional Probability and the Rules of Probability, and Using Probability to Make Decisions. Immediately following the domain and enclosed in brackets is an abbreviation denoting the domain. Identified below each domain are the clusters that serve to group related content standards.

The Standards for Mathematical Practice are listed below in the column to the right. These mathematical practice standards should be incorporated into classroom instruction of the content standards.

Content Standard Domains and Clusters

Interpreting Categorical and Quantitative Data [S-ID]
- Summarize, represent, and interpret data on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models.

Making Inferences and Justifying Conclusions [S-IC]
- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Conditional Probability and the Rules of Probability [S-CP]
- Understand independence and conditional probability and use them to interpret data.
- Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Using Probability to Make Decisions [S-MD]
- Calculate expected values and use them to solve problems.
- Use probability to evaluate outcomes of decisions.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ALGEBRA I

The Algebra I course builds on foundational mathematics content learned by students in Grades K-8 by expanding mathematics understanding to provide students with a strong mathematics education. Content is designed to engage students in a variety of mathematical experiences that include the use of reasoning and problem-solving skills, which may be applied to life situations beyond the classroom setting. This course serves as the cornerstone for all high school mathematics courses; therefore, all subsequent mathematics courses require student mastery of the Algebra I content standards.

Algebra I is one of the courses required for all students. School systems may offer Algebra I and Algebra IA and Algebra IB. Content standards in Algebra IA should focus on rational numbers, arithmetic sequences, and linear functions. Standards 3, 4, 5, 6, 7, 7a, 7b, 8, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 28, 29, 30, 31, 31a, 32, 33, 34, 34a, 34b, 35, 36, 37a, 37b, 38, 40, 45, 45a, 45b, 45c, and 46 must be taught in the Algebra IA course. Content standards in Algebra IB should focus on irrational numbers, geometric sequences, and quadratic and exponential functions. Standards 1, 2, 3, 7, 7a, 7b, 8, 9, 9a, 9b, 9c, 9d, 10, 11, 12, 13, 16, 18, 18a, 18b, 21, 23, 27, 28, 29, 30, 31, 31a, 31b, 32, 32a, 32b, 33, 34, 34a, 34b, 35, 36, 37, 37a, 37c, 38, 39, 40, 41, 42, 43, 44, 45, 45a, 45b, and 47 must be taught in the Algebra IB course. Systems offering Algebra I in the eighth grade have the responsibility of ensuring that all Algebra I course content standards and Grade 8 course content standards be included in instruction.

Students will:

NUMBER AND QUANTITY

The Real Number System

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. [N-RN1]
   Example: We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. [N-RN2]

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. [N-RN3]

Quantities*

Reason quantitatively and use units to solve problems. (*Foundation for work with expressions, equations, and functions.)*

4. Use units as a way to understand problems and to guide the solution of multistep problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. [N-Q1]

5. Define appropriate quantities for the purpose of descriptive modeling. [N-Q2]

6. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. [N-Q3]

ALGEBRA

Seeing Structure in Expressions

Interpret the structure of expressions. (*For standard 7 linear, exponential, quadratic; for standard 8 linear, exponential, quadratic, rational.)*

7. Interpret expressions that represent a quantity in terms of its context.* [A-SSE1]
   a. Interpret parts of an expression such as terms, factors, and coefficients. [A-SSE1a]
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. [A-SSE1b]
      Example: Interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

8. Use the structure of an expression to identify ways to rewrite it. [A-SSE2]
   Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems. *(Quadratic and exponential.)*

9. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* [A-SSE3]
   a. Factor a quadratic expression to reveal the zeros of the function it defines. [A-SSE3a]
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. [A-SSE3b]
   c. Determine a quadratic equation when given its graph or roots.
   d. Use the properties of exponents to transform expressions for exponential functions. [A-SSE3c]
      Example: The expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Arithmetic With Polynomials and Rational Expressions

Perform arithmetic operations on polynomials. *(Linear and quadratic.)*

10. Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. [A-APR1]

Rewrite rational expressions. ***(Linear and quadratic denominators.)***

11. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. [A-APR7]

Creating Equations*

Create equations that describe numbers or relationships. ***(Linear, quadratic, and exponential (integer inputs only); for Standard 14, linear only.)***

12. Create equations and inequalities in one variable, and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* [A-CED1]

13. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [A-CED2]

14. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context. [A-CED3]
    Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods.

15. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. [A-CED4]
    Example: Rearrange Ohm's law $V = IR$ to highlight resistance R.

Reasoning With Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning. ***(Master linear; learn as general principle.)***

16. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. [A-REI1]

Solve equations and inequalities in one variable. ***(Linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions.)***

17. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. [A-REI3]

18. Solve quadratic equations in one variable. [A-REI4]
    a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. [A-REI4a]
    b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square and the quadratic formula, and factoring as appropriate to the initial form of the equation.[A-REI4b]

Solve systems of equations. ***(Linear-linear and linear-quadratic.)***

19. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. [A-REI5]

20. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. [A-REI6]

21. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. [A-REI7]
    Example: Find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$.

Represent and solve equations and inequalities graphically. ***(Linear and exponential; learn as general principle.)***

22. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). [A-REI10]

23. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* [A-REI11]

24. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. [A-REI12]

FUNCTIONS

Interpreting Functions

Understand the concept of a function and use function notation. ***(Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences.)***

25. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. [F-IF1]

26. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. [F-IF2]

27. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. [F-IF3]
    Example: The Fibonacci sequence is defined recursively by
    $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

Interpret functions that arise in applications in terms of the context. (*Linear, exponential, and quadratic.*)

28. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.* [F-IF4]

29. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.* [F-IF5]
    Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

30. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* [F-IF6]

Analyze functions using different representations. *(Linear, exponential, quadratic, absolute value, step, and an awareness of piecewise-defined.)*

31. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* [F-IF7]
    a. Graph linear and quadratic functions, and show intercepts, maxima, and minima. [F-IF7a]
    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [F-IF7b]

32. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. [F-IF8]
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. [F-IF8a]
    b. Use the properties of exponents to interpret expressions for exponential functions. [F-IF8b]
       Example: Identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, and $y = (1.2)^{t/10}$, and classify them as representing exponential growth and decay.

33. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). [F-IF9]
    Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

Build a function that models a relationship between two quantities. ***(For standards 34 and 35, linear, exponential, and quadratic.)***

34. Write a function that describes a relationship between two quantities.* [F-BF1]
    a. Determine an explicit expression, a recursive process, or steps for calculation from a context. [F-BF1a]
    b. Combine standard function types using arithmetic operations. [F-BF1b]
        Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

35. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.* [F-BF2]

Build new functions from existing functions. ***(Linear, exponential, quadratic, and absolute value.)***

36. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. [F-BF3]

Linear, Quadratic, and Exponential Models*

Construct and compare linear, quadratic, and exponential models and solve problems.

37. Distinguish between situations that can be modeled with linear functions and with exponential functions. [F-LE1]
    a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. [F-LE1a]
    b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. [F-LE1b]
    c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. [F-LE1c]

38. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). [F-LE2]

39. Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. [F-LE3]

Interpret expressions for functions in terms of the situation they model. ***(Linear and exponential of form $f(x) = b^x + k$.)***

40. Interpret the parameters in a linear or exponential function in terms of a context. [F-LE5]

STATISTICS AND PROBABILITY

Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

41. Represent data with plots on the real number line (dot plots, histograms, and box plots). [S-ID1]

42. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. [S-ID2]

43. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). [S-ID3]

Summarize, represent, and interpret data on two categorical and quantitative variables. ***(Linear focus, discuss general principle.)***

44. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. [S-ID5]

45. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. [S-ID6]
    a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* [S-ID6a]
    b. Informally assess the fit of a function by plotting and analyzing residuals. [S-ID6b]
    c. Fit a linear function for a scatter plot that suggests a linear association. [S-ID6c]

Interpret linear models.

46. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. [S-ID7]

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data.
(Link to data from simulations or experiments.)

47. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. [S-CP2]

GEOMETRY

The Geometry course builds on Algebra I concepts and increases students' knowledge of shapes and their properties through geometry-based applications, many of which are observable in aspects of everyday life. This knowledge helps develop visual and spatial sense and strong reasoning skills. The Geometry course requires students to make conjectures and to use reasoning to validate or negate these conjectures. The use of proofs and constructions is a valuable tool that enhances reasoning skills and enables students to better understand more complex mathematical concepts. Technology should be used to enhance students' mathematical experience, not replace their reasoning abilities. Because of its importance, this Euclidean geometry course is required of all students receiving an Alabama High School Diploma.

School systems may offer Geometry and Geometry A and Geometry B. Content standards 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 26, 30, 31, 32, 33, and 34 must be taught in the Geometry A course. Content standards 2, 12, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 35, 36, 37, 38, 39, 40, 41, 42, and 43 must be taught in the Geometry B course.

Students will:

GEOMETRY

Congruence

Experiment with transformations in the plane.

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment based on the undefined notions of point, line, distance along a line, and distance around a circular arc. [G-CO1]

2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). [G-CO2]

3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. [G-CO3]

4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. [G-CO4]

5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. [G-CO5]

Understand congruence in terms of rigid motions. ***(Build on rigid motions as a familiar starting point for development of concept of geometric proof.)***

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. [G-CO6]

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. [G-CO7]

8. Explain how the criteria for triangle congruence, angle-side-angle (ASA), side-angle-side (SAS), and side-side-side (SSS), follow from the definition of congruence in terms of rigid motions. [G-CO8]

Prove geometric theorems. ***(Focus on validity of underlying reasoning while using variety of ways of writing proofs.)***

9. Prove theorems about lines and angles. *Theorems include vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; and points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* [G-CO9]

10. Prove theorems about triangles. *Theorems include measures of interior angles of a triangle sum to 180º, base angles of isosceles triangles are congruent, the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length, and the medians of a triangle meet at a point.* [G-CO10]

11. Prove theorems about parallelograms. *Theorems include opposite sides are congruent, opposite angles are congruent; the diagonals of a parallelogram bisect each other; and conversely, rectangles are parallelograms with congruent diagonals.* [G-CO11]

Make geometric constructions. ***(Formalize and explain processes.)***

12. Make formal geometric constructions with a variety of tools and methods such as compass and straightedge, string, reflective devices, paper folding, and dynamic geometric software. *Constructions include copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* [G-CO12]

13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. [G-CO13]

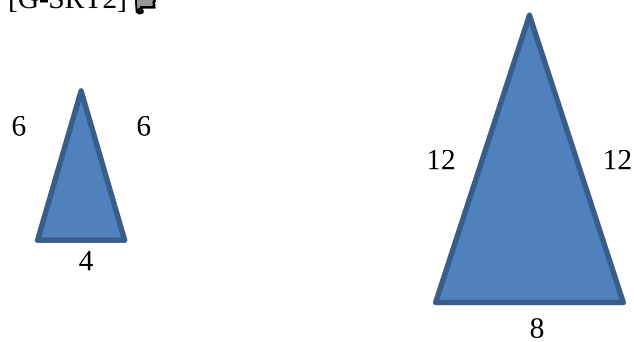
Similarity, Right Triangles, and Trigonometry

Understand similarity in terms of similarity transformations.

14. Verify experimentally the properties of dilations given by a center and a scale factor. [G-SRT1]
    a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. [G-SRT1a]
    b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. [G-SRT1b]

15. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. [G-SRT2]

    Example 1:



Given the two triangles above, show that they are similar.

$$\frac{4}{8} = \frac{6}{12}$$

They are similar by SSS. The scale factor is equivalent.

    Example 2:

Show that the triangles are similar.
Two corresponding sides are proportional and the included angle is congruent.
(SAS similarity)

16. Use the properties of similarity transformations to establish the angle-angle (AA) criterion for two triangles to be similar. [G-SRT3]

Prove theorems involving similarity.

17. Prove theorems about triangles. *Theorems include a line parallel to one side of a triangle divides the other two proportionally, and conversely; and the Pythagorean Theorem proved using triangle similarity.* [G-SRT4]

18. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. [G-SRT5]

Define trigonometric ratios and solve problems involving right triangles.

19. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle leading to definitions of trigonometric ratios for acute angles. [G-SRT6]

20. Explain and use the relationship between the sine and cosine of complementary angles. [G-SRT7]

21. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.* [G-SRT8]

Apply trigonometry to general triangles.

22. (+) Prove the Law of Sines and the Law of Cosines and use them to solve problems. [G-SRT10]

23. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). [G-SRT11]

Circles

Understand and apply theorems about circles.

24. Prove that all circles are similar. [G-C1]

25. Identify and describe *relationships* among inscribed angles, radii, and chords. *Include the* relationship *between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* [G-C2]

26. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. [G-C3]

27. (+) Construct a tangent line from a point outside a given circle to the circle. [G-C4]

Find arc lengths and areas of sectors of circles. ***(Radian introduced only as unit of measure.)***

28. Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. [G-C5]

Expressing Geometric Properties With Equations

Translate between the geometric description and the equation for a conic section.

29. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. [G-GPE1]

Use coordinates to prove simple geometric theorems algebraically. ***(Include distance formula; relate to Pythagorean Theorem.)***

30. Use coordinates to prove simple geometric theorems algebraically. [G-GPE4]
    Example: Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).

31. Prove the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). [G-GPE5]

32. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. [G-GPE6]

33. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.* [G-GPE7]

Use coordinates to prove simple geometric theorems algebraically.

34. Determine areas and perimeters of regular polygons, including inscribed or circumscribed polygons, given the coordinates of vertices or other characteristics.

Geometric Measurement and Dimension

Explain volume formulas and use them to solve problems.

35. Give an informal argument for the formulas for the circumference of a circle; area of a circle; and volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* [G-GMD1]

36. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.* [G-GMD3]

37. Determine the relationship between surface areas of similar figures and volumes of similar figures.

Visualize relationships between two-dimensional and three-dimensional objects.

38. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. [G-GMD4]

Modeling With Geometry

Apply geometric concepts in modeling situations.

39. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* [G-MG1]

40. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, British Thermal Units (BTUs) per cubic foot).* [G-MG2]

41. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost, working with typographic grid systems based on ratios).* [G-MG3]

STATISTICS AND PROBABILITY

Using Probability to Make Decisions

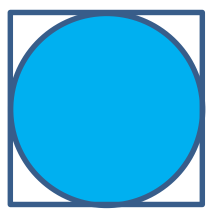
Use probability to evaluate outcomes of decisions. ***(Introductory; apply counting rules.)***

42. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). [S-MD6]

43. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). [S-MD7]

Example:

6 units



What is the probability of tossing a penny and having it land in the non-shaded region?
Geometric Probability is the Non-Shaded Area divided by the Total Area.

$$\frac{(6^2)-\pi(3^2)}{6^2}=\frac{36-9\pi}{36}=\frac{4-\pi}{4} \text{ or } 1-\frac{\pi}{4}$$

ALGEBRA II WITH TRIGONOMETRY

Algebra II With Trigonometry is a course designed to extend students' knowledge of Algebra I with additional algebraic and trigonometric content. Mastery of the content standards for this course is necessary for student success in higher-level mathematics. The use of appropriate technology is encouraged for numerical and graphical investigations that enhance analytical comprehension.

Algebra II With Trigonometry is required for all students pursuing the Alabama High School Diploma with Advanced Academic Endorsement. Prerequisites for this course are Algebra I and Geometry. If a student chooses to take the Algebraic Connections course, it must be taken prior to the Algebra II With Trigonometry course.

Students will:

NUMBER AND QUANTITY

The Complex Number System

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. [N-CN1]

2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. [N-CN2]

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. [N-CN3]

Use complex numbers in polynomial identities and equations. *(Polynomials with real coefficients.)*

4. Solve quadratic equations with real coefficients that have complex solutions. [N-CN7]

5. (+) Extend polynomial identities to the complex numbers.
   Example: Rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$. [N-CN8]

6. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. [N-CN9]

Vector and Matrix Quantities

Perform operations on matrices and use matrices in applications.

7. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. (*Use technology to approximate roots.)* [N-VM6]

8. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. [N-VM7]

9. (+) Add, subtract, and multiply matrices of appropriate dimensions. [N-VM8]

10. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. [N-VM9]

11. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. [N-VM10]

ALGEBRA

Seeing Structure in Expressions

Interpret the structure of expressions. (*Polynomial and rational.)*

12. Interpret expressions that represent a quantity in terms of its context.* [A-SSE1]
    a. Interpret parts of an expression such as terms, factors, and coefficients. [A-SSE1a]
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. [A-SSE1b]
        Example: Interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

13. Use the structure of an expression to identify ways to rewrite it. [A-SSE2]
    Example: See $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

14. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.* [A-SSE4]
    Example: Calculate mortgage payments.

Arithmetic With Polynomials and Rational Expressions

Perform arithmetic operations on polynomials. *(Beyond quadratic.)*

15. Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. [A-APR1]

Understand the relationship between zeros and factors of polynomials.

16. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$. [A-APR2]

17. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. [A-APR3]

Use polynomial identities to solve problems.

18. Prove polynomial identities and use them to describe numerical relationships. [A-APR4]
    Example: The polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

Rewrite rational expressions. ***(Linear and quadratic denominators.)***

19. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system. [A-APR6]

Creating Equations*

Create equations that describe numbers or relationships. ***(Equations using all available types of expressions, including simple root functions.)***

20. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. [A-CED1]

21. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [A-CED2]

22. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. [A-CED3]
    Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods.

23. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. [A-CED4]
    Example: Rearrange Ohm's law $V = IR$ to highlight resistance R.

Reasoning With Equations and Inequalities

Understand solving equations as a process of reasoning, and explain the reasoning. ***(Simple rational and radical.)***

24. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. [A-REI2]

Solve equations and inequalities in one variable.

25. Recognize when the quadratic formula gives complex solutions, and write them as $a \pm bi$ for real numbers a and b. [A-REI4b]

Solve systems of equations.

26. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). [A-REI9]

Represent and solve equations and inequalities graphically. ***(Combine polynomial, rational, radical, absolute value, and exponential functions.)***

27. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* [A-REI11]

Conic Sections

Understand the graphs and equations of conic sections. ***(Emphasize understanding graphs and equations of circles and parabolas.)***

28. Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.
    Example: Graph $x^2 - 6x + y^2 - 12y + 41 = 0$ or $y^2 - 4x + 2y + 5 = 0$.

    a. Formulate equations of conic sections from their determining characteristics.
       Example: Write the equation of an ellipse with center (5, -3), a horizontal major axis of length 10, and a minor axis of length 4.

       Answer: $\frac{(x-5)^2}{25} + \frac{(y+3)^2}{4} = 1.$

FUNCTIONS

Interpreting Functions

Interpret functions that arise in applications in terms of the context. ***(Emphasize selection of appropriate models.)***

29. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.* [F-IF5]
    Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Analyze functions using different representations. ***(Focus on using key features to guide selection of appropriate type of model function.)***

30. Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* [F-IF7]
    a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [F-IF7b]
    b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. [F-IF7c]
    c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. [F-IF7e]

31. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. [F-IF8]
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. [F-IF8a]

       Example: $f(x) = x^2 + 6x + 5$. Locate the vertex, axis of symmetry, show the zeros and extreme values.

    b. Use the properties of exponents to interpret expressions for exponential functions. [F-IF8b]

       Example: Identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, and y = (1.2)t/10, and classify them as representing exponential growth and decay.

32. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). [F-IF9]

    Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

Build a function that models a relationship between two quantities. ***(Include all types of functions studied.)***

33. Write a function that describes a relationship between two quantities.* [F-BF1]
    a. Combine standard function types using arithmetic operations. [F-BF1b]

       Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Build new functions from existing functions. ***(Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types.)***

34. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include

recognizing even and odd functions from their graphs and algebraic expressions for them. [F-BF3]

35. Find inverse functions. [F-BF4]
    a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse, and write an expression for the inverse. [F-BF4a]
       Example: $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

Linear, Quadratic, and Exponential Models*

Construct and compare linear, quadratic, and exponential models and solve problems. *(Logarithms as solutions for exponentials.)*

36. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers, and the base b is 2, 10, or e; evaluate the logarithm using technology. [F-LE4]

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

37. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. [F-TF1]

38. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. [F-TF2]

39. Define the six trigonometric functions using ratios of the sides of a right triangle, coordinates on the unit circle, and the reciprocal of other functions.

Model periodic phenomena with trigonometric functions.

40. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* [F-TF5]

STATISTICS AND PROBABILITY

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions. (*Include more complex situations*.)

41. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). [S-MD6]

42. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). [S-MD7]

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data. ***(Link to data from simulations or experiments.)***

43. Describe events as subsets of a sample space (the set of outcomes), using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). [S-CP1]

44. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. [S-CP3]

45. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. [S-CP4]

    Example: Collect data from a random sample of students in your school on their favorite subject among mathematics, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

46. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. [S-CP5]

    Example: Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

47. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. [S-CP6]

48. Apply the Addition Rule, *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) - *P*(*A* and *B*), and interpret the answer in terms of the model. [S-CP7]

49. (+) Apply the general Multiplication Rule in a uniform probability model, *P*(*A* and *B*) = $P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. [S-CP8]

50. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. [S-CP9]

PRECALCULUS

Precalculus is a course designed for students who have successfully completed the Algebra II With Trigonometry course. This course is considered to be a prerequisite for success in calculus and college mathematics. Algebraic, graphical, numerical, and verbal analyses are incorporated during investigations of the Precalculus content standards. Parametric equations, polar relations, vector operations, and limits are introduced. Content for this course also includes an expanded study of polynomial and rational functions, conic sections, trigonometric functions, and logarithmic and exponential functions.

Application-based problem solving is an integral part of the course. Instruction should include appropriate use of technology to facilitate continued development of students' higher-order thinking skills.

Students will:

NUMBER AND QUANTITY

The Complex Number System

Represent complex numbers and their operations on the complex plane.

1. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. [N-CN4]

2. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. [N-CN5]

   Example: $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.

3. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. [N-CN6]

Limits

Understand limits of functions.

4. Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity.
   a. Apply limits in problems involving convergence and divergence.

Vector and Matrix Quantities

Represent and model with vector quantities.

5. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, v). [N-VM1]

6. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. [N-VM2]

7. (+) Solve problems involving velocity and other quantities that can be represented by vectors. [N-VM3]

Perform operations on vectors.

8. (+) Add and subtract vectors. [N-VM4]
   a. (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. [N-VM4a]
   b. (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. [N-VM4b]
   c. (+) Understand vector subtraction **v** – **w** as **v** + (–**w**), where –**w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. [N-VM4c]

9. (+) Multiply a vector by a scalar. [N-VM5]
   a. (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. [N-VM5a]
   b. (+) Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|c\mathbf{v}\| = |c|$v. Compute the direction of $c\mathbf{v}$ knowing that when $|c|$v $\neq 0$, the direction of $c\mathbf{v}$ is either along **v** (for $c > 0$) or against **v** (for $c < 0$). [N-VM5b]

Perform operations on matrices and use matrices in applications.

10. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. [N-VM11]

11. (+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. [N-VM12]

ALGEBRA

Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems.

12. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.* *(Extend to infinite geometric series.)* [A-SSE4]
    Example: Calculate mortgage payments.

Arithmetic With Polynomials and Rational Expressions

Use polynomial identities to solve problems.

13. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.) [A-APR5]

Reasoning With Equations and Inequalities

Solve systems of equations.

14. (+) Represent a system of linear equations as a single matrix equation in a vector variable. [A-REI8]

Conic Sections

Understand the graphs and equations of conic sections.

15. Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.
    Example: Graph $x^2 - 6x + y^2 - 12y + 41 = 0$ or $y^2 - 4x + 2y + 5 = 0$.

    a. Formulate equations of conic sections from their determining characteristics.
       Example: Write the equation of an ellipse with center (5, -3), a horizontal major axis of length 10, and a minor axis of length 4.

       Answer: $\frac{(x-5)^2}{25} + \frac{(y+3)^2}{4} = 1.$

FUNCTIONS

Interpreting Functions

Interpret functions that arise in applications in terms of the context. ***(Emphasize selection of appropriate models. Understand limits of functions.)***

16. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (*Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Determine odd, even, neither.)** [F-IF4]

17. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* [F-IF6]

Analyze functions using different representations. ***(Focus on using key features to guide selection of appropriate type of model function with emphasis on piecewise, step, and absolute value. Also emphasize inverse and transformations of polynomials, rational, radical, absolute value, and trigonometric functions.)***

18. Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* [F-IF7]
    a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [F-IF7b]
    b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. [F-IF7c]
    c. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. [F-IF7d]
    d. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. [F-IF7e]

Building Functions

Build a function that models a relationship between two quantities.

19. (+) Compose functions. [F-BF1c]
    Example: If $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

Build new functions from existing functions.

20. Determine the inverse of a function and a relation.

21. (+) Verify by composition that one function is the inverse of another. [F-BF4b]

22. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. [F-BF4c]

23. (+) Produce an invertible function from a non-invertible function by restricting the domain. [F-BF4d]

24. (+) Understand the inverse relationship between exponents and logarithms, and use this relationship to solve problems involving logarithms and exponents. [F-BF5]

25. Compare effects of parameter changes on graphs of transcendental functions.
    Example: Explain the relationship of the graph $y = e^{x-2}$ to the graph $y = e^x$.

Trigonometric Functions

Recognize attributes of trigonometric functions and solve problems involving trigonometry.

26. Determine the amplitude, period, phase shift, domain, and range of trigonometric functions and their inverses.

27. Use the sum, difference, and half-angle identities to find the exact value of a trigonometric function.

28. Utilize parametric equations by graphing and by converting to rectangular form.
    a. Solve application-based problems involving parametric equations.
    b. Solve applied problems that include sequences with recurrence relations.

Extend the domain of trigonometric functions using the unit circle.

29. (+) Use special triangles to determine geometrically the values of sine, cosine, and tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. [F-TF3]

30. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. [F-TF4]

Model periodic phenomena with trigonometric functions.

31. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. [F-TF6]

32. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.* [F-TF7]

Prove and apply trigonometric identities.

33. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. [F-TF8]

34. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems. [F-TF9]

GEOMETRY

Similarity, Right Triangles, and Trigonometry

Apply trigonometry to general triangles.

35. (+) Derive the formula $A = (\frac{1}{2})ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. (*Apply formulas previously derived in Geometry*.) [G-SRT9]

Expressing Geometric Properties With Equations

Translate between the geometric description and the equation for a conic section.

36. (+) Derive the equations of a parabola given a focus and directrix. [G-GPE2]

37. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. [G-GPE3]

Explain volume formulas and use them to solve problems.

38. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. [G-GMD2]

STATISTICS AND PROBABILITY

Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

39. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. *(Focus on increasing rigor using standard deviation.)* [S-ID2]

40. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). *(Identify uniform, skewed, and normal distributions in a set of data. Determine the quartiles and interquartile range for a set of data.)* [S-ID3]

41. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. [S-ID4]

Interpret linear models.

42. Compute (using technology) and interpret the correlation coefficient of a linear fit. [S-ID8]

43. Distinguish between correlation and causation. [S-ID9]

Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

44. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. [S-IC1]

45. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. [S-IC2]
    Example: A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

46. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. [S-IC3]

47. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. [S-IC4]

48. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. [S-IC5]

49. Evaluate reports based on data. [S-IC6]

Using Probability to Make Decisions

Calculate expected values and use them to solve problems.

50. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. [S-MD1]

51. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. [S-MD2]

52. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. [S-MD3]
    Example: Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.

53. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. [S-MD4]

    Example: Find a current data distribution on the number of television sets per household in the United States, and calculate the expected number of sets per household. How many television sets would you expect to find in 100 randomly selected households?

Use probability to evaluate outcomes of decisions.

54. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. [S-MD5]
    a. Find the expected payoff for a game of chance. [S-MD5a]

       Examples: Find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.
    b. Evaluate and compare strategies on the basis of expected values. [S-MD5b]

       Example: Compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.

TABLE 1*

ADDITION AND SUBTRACTION SITUATIONS

SITUATION	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

SITUATION	Total Unknown	Addend Unknown	Both Addends Unknown (These take-apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in, but always does mean is the same number as.)
Put Together/ Take Apart (Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.)	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

SITUATION	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare (For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.)	(Version with "How many more?"): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (Version with "How many fewer?"): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

*Adapted from *Mathematics Learning in Early Childhood: Paths Towards Excellence and Equity*, National Research Council 2009, pp. 32-33.

TABLE 2

MULTIPLICATION AND DIVISION SITUATIONS

(The first example in each cell is an example of discrete things. These are easier for students and should be given before the measurement examples.)			
SITUATION	**Unknown Product** **3 x 6 = ?**	**Group Size Unknown** **("How many in each group?" Division)** **3 x ? = 18, and 18 ÷ 3 = ?**	**Number of Groups Unknown** **("How many groups?" Division)** **? x 6 = 18, and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example:* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example:* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*: You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays (The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.) **Area** (Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.)	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*: What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example:* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*: A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example:* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example:* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

TABLE 3

PROPERTIES OF OPERATIONS

Here *a*, *b*, and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.	
Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

TABLE 4

PROPERTIES OF EQUALITY

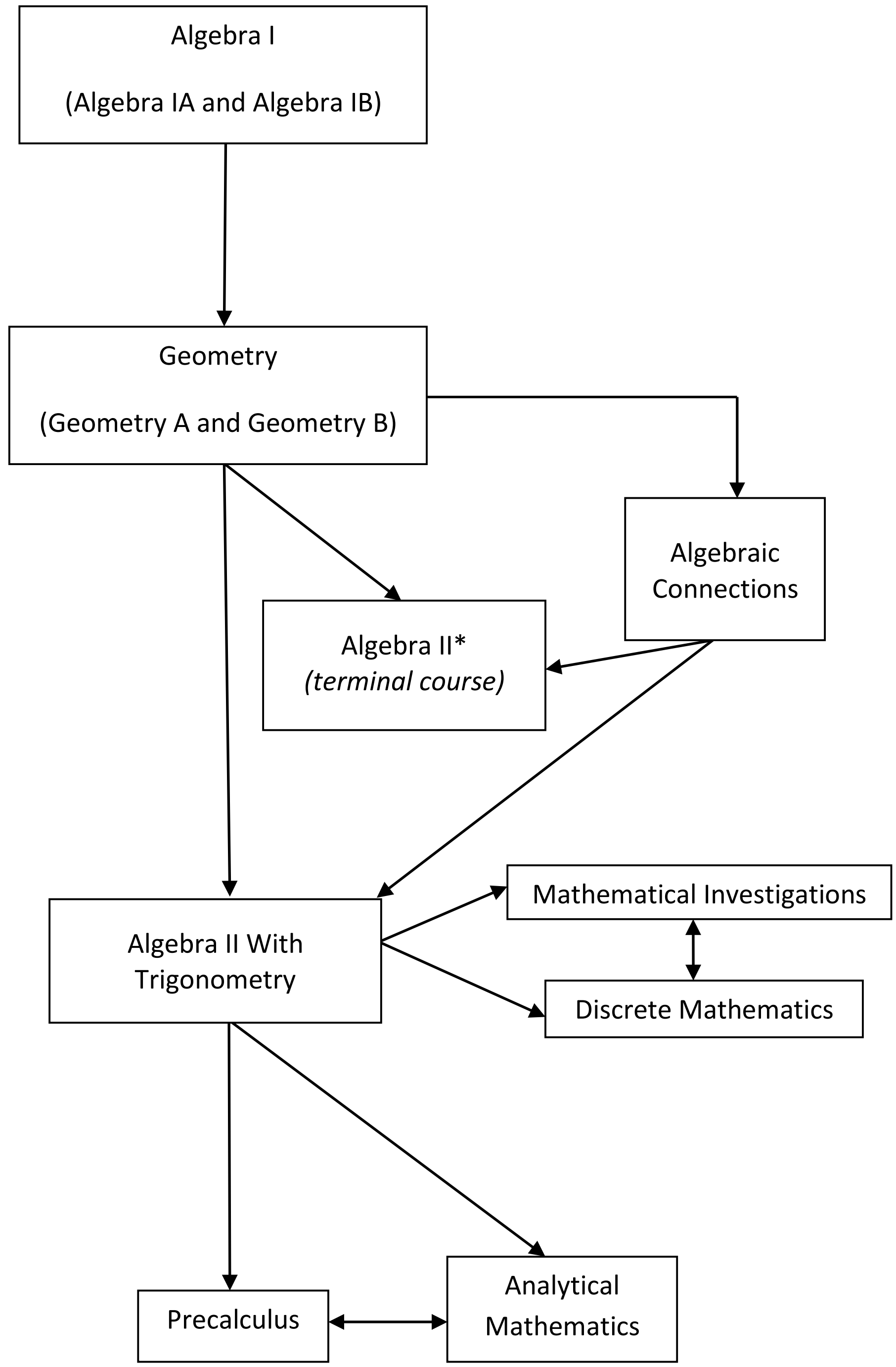
Here *a*, *b*, and *c* stand for arbitrary numbers in the rational, real, or complex number systems.	
Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

TABLE 5

PROPERTIES OF INEQUALITY

Here *a*, *b*, and *c* stand for arbitrary numbers in the rational or real number systems.
Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

POSSIBLE COURSE PROGRESSION IN GRADES 9-12



Algebra II or Algebra II With Trigonometry is required for all students pursuing the Alabama High School Diploma or the Alabama High School Diploma with Advanced Academic Endorsement, respectively.

*Algebra II, Algebra II with Trigonometry, or an ALSDE-approved substitute is required for all students pursuing the Alabama High School Diploma

POSSIBLE COURSE PATHWAYS

There are several pathways by which a student can meet the high school graduation requirements for earning four credits in mathematics in Grades 9-12. Local school systems may determine which pathways lead to completion of the requirements for a specific diploma, provided the minimum requirements set forth by the Alabama State Board of Education are followed. Some pathways in Grades 9-12 are indicated below.

Pathways for Students Who Begin Algebra I in Grade 9

Algebra I
Geometry
Algebra II With Trigonometry
Precalculus

Algebra I
Geometry
Algebra II With Trigonometry
Analytical Mathematics

Algebra I
Geometry
Algebra II With Trigonometry
Discrete Mathematics

Algebra I
Geometry
Algebra II With Trigonometry
Mathematical Investigations

Algebra I
Geometry
Algebraic Connections
Algebra II With Trigonometry

Algebra IA
Algebra IB
Geometry
Algebra II With Trigonometry

Algebra I
Geometry A
Geometry B
Algebra II With Trigonometry

Algebra I
Geometry
Algebraic Connections
Algebra II

Algebra IA
Algebra IB
Geometry
Algebra II

Algebra I
Geometry A
Geometry B
Algebra II

Some Pathways for Students Who Complete Algebra I in Grade 8

Geometry
Algebra II With Trigonometry
Precalculus
Analytical Mathematics

Geometry
Algebra II With Trigonometry
Analytical Mathematics
Precalculus

Geometry
Algebra II With Trigonometry
Precalculus
Advanced Placement (AP) Mathematics Course

Geometry
Algebraic Connections
Algebra II With Trigonometry
Analytical Mathematics

Geometry
Algebra II With Trigonometry
Discrete Mathematics
Precalculus

Geometry
Algebra II With Trigonometry
Mathematical Investigations
Precalculus

Geometry
Algebra II With Trigonometry
Mathematical Investigations
Discrete Mathematics

Geometry
Algebra II With Trigonometry
Discrete Mathematics
Mathematical Investigations

Geometry
Algebraic Connections
Algebra II With Trigonometry
Precalculus

Geometry
Algebraic Connections
Algebra II With Trigonometry
Analytical Mathematics

Geometry
Algebraic Connections
Algebra II With Trigonometry
Discrete Mathematics

Geometry
Algebraic Connections
Algebra II With Trigonometry
Mathematical Investigations

LITERACY STANDARDS FOR GRADES 6-12:
HISTORY/SOCIAL STUDIES, SCIENCE, AND TECHNICAL SUBJECTS

College and Career Readiness Anchor Standards for Reading

The Grades 6-12 standards on the following pages define what students should understand and be able to do by the end of each grade span. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Key Ideas and Details

1. Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
2. Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
3. Analyze how and why individuals, events, or ideas develop and interact over the course of a text.

Craft and Structure

4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
5. Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
6. Assess how point of view or purpose shapes the content and style of a text.

Integration of Knowledge and Ideas

7. Integrate and evaluate content presented in diverse formats and media, including visually and quantitatively, as well as in words.*
8. Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
9. Analyze how two or more texts address similar themes or topics in order to build knowledge or to compare the approaches the authors take.

Range of Reading and Level of Text Complexity

10. Read and comprehend complex literary and informational texts independently and proficiently.

*See College And Career Readiness Anchor Standards for Writing, "Research to Build and Present Knowledge," on page 134 for additional standards relevant to gathering, assessing, and applying information from print and digital sources.

Reading Standards for Literacy in History/Social Studies 6–12

The standards below begin at Grade 6; standards for K-5 reading in history/social studies, science, and technical subjects are integrated into the K-5 Reading standards. The CCR anchor standards and high school standards in literacy work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 6-8 Students:	Grades 9-10 Students:	Grades 11-12 Students:
Key Ideas and Details		
1. Cite specific textual evidence to support analysis of primary and secondary sources.	1. Cite specific textual evidence to support analysis of primary and secondary sources, attending to such features as the date and origin of the information.	1. Cite specific textual evidence to support analysis of primary and secondary sources, connecting insights gained from specific details to an understanding of the text as a whole.
2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary of the source distinct from prior knowledge or opinions.	2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary of how key events or ideas develop over the course of the text.	2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary that makes clear the relationships among the key details and ideas.
3. Identify key steps in a text's description of a process related to history/social studies (e.g., how a bill becomes law, how interest rates are raised or lowered).	3. Analyze in detail a series of events described in a text; determine whether earlier events caused later ones or simply preceded them.	3. Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain.
Craft and Structure		
4. Determine the meaning of words and phrases as they are used in a text, including vocabulary specific to domains related to history/social studies.	4. Determine the meaning of words and phrases as they are used in a text, including vocabulary describing political, social, or economic aspects of history/social studies.	4. Determine the meaning of words and phrases as they are used in a text, including analyzing how an author uses and refines the meaning of a key term over the course of a text (e.g., how Madison defines *faction* in *Federalist* No. 10).
5. Describe how a text presents information (e.g., sequentially, comparatively, causally).	5. Analyze how a text uses structure to emphasize key points or advance an explanation or analysis.	5. Analyze in detail how a complex primary source is structured, including how key sentences, paragraphs, and larger portions of the text contribute to the whole.
6. Identify aspects of a text that reveal an author's point of view or purpose (e.g., loaded language, inclusion or avoidance of particular facts).	6. Compare the point of view of two or more authors for how they treat the same or similar topics, including which details they include and emphasize in their respective accounts.	6. Evaluate authors' differing points of view on the same historical event or issue by assessing the authors' claims, reasoning, and evidence.
Integration of Knowledge and Ideas		
7. Integrate visual information (e.g., in charts, graphs, photographs, videos, or maps) with other information in print and digital texts.	7. Integrate visual information (e.g., in charts).	7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, as well as in words) in order to address a question or solve a problem.
8. Distinguish among fact, opinion, and reasoned judgment in a text.	8. Assess the extent to which the reasoning and evidence in a text support the author's claims.	8. Evaluate an author's premises, claims, and evidence by corroborating or challenging them with other information.
9. Analyze the relationship between a primary and secondary source on the same topic.	9. Compare and contrast treatments of the same topic in several primary and secondary sources.	9. Integrate information from diverse sources, both primary and secondary, into a coherent understanding of an idea or event, noting discrepancies among sources.
Range of Reading and Level of Text Complexity		
10. By the end of Grade 8, read and comprehend history/social studies texts in the Grades 6-8 text complexity band independently and proficiently.	10. By the end of Grade 10, read and comprehend history/social studies texts in the Grades 9-10 text complexity band independently and proficiently.	10. By the end of Grade 12, read and comprehend history/social studies texts in the Grades 11-CCR text complexity band independently and proficiently.

Reading Standards for Literacy in Science and Technical Subjects 6–12

Grades 6-8 Students:	Grades 9-10 Students:	Grades 11-12 Students:
Key Ideas and Details		
1. Cite specific textual evidence to support analysis of science and technical texts.	1. Cite specific textual evidence to support analysis of science and technical texts, attending to the precise details of explanations or descriptions.	1. Cite specific textual evidence to support analysis of science and technical texts, attending to important distinctions the author makes and to any gaps or inconsistencies in the account.
2. Determine the central ideas or conclusions of a text; provide an accurate summary of the text distinct from prior knowledge or opinions.	2. Determine the central ideas or conclusions of a text; trace the text's explanation or depiction of a complex process, phenomenon, or concept; provide an accurate summary of the text.	2. Determine the central ideas or conclusions of a text; summarize complex concepts, processes, or information presented in a text by paraphrasing them in simpler but still accurate terms.
3. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.	3. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.	3. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
Craft and Structure		
4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *Grades 6-8 texts and topics.*	4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *Grades 9-10 texts and topics.*	4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *Grades 11-12 texts and topics.*
5. Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to an understanding of the topic.	5. Analyze the structure of the relationships among concepts in a text, including relationships among key terms (e.g., force, friction, reaction force, energy).	5. Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.
6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text.	6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, defining the question the author seeks to address.	6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, identifying important issues that remain unresolved.
Integration of Knowledge and Ideas		
7. Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).	7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.	7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.
8. Distinguish among facts, reasoned judgment based on research findings, and speculation in a text.	8. Assess the extent to which the reasoning and evidence in a text support the author's claim or a recommendation for solving a scientific or technical problem.	8. Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.
9. Compare and contrast the information gained from experiments, simulations, video, or multimedia sources with that gained from reading a text on the same topic.	9. Compare and contrast findings presented in a text to those from other sources (including their own experiments), noting when the findings support or contradict previous explanations or accounts.	9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.
Range of Reading and Level of Text Complexity		
10. By the end of Grade 8, read and comprehend science/technical texts in the Grades 6-8 text complexity band independently and proficiently.	10. By the end of Grade 10, read and comprehend science/technical texts in the Grades 9-10 text complexity band independently and proficiently.	10. By the end of Grade 12, read and comprehend science/technical texts in the Grades 11-CCR text complexity band independently and proficiently.

College and Career Readiness Anchor Standards for Writing

The Grades 6-12 standards on the following pages define what students should understand and be able to do by the end of each grade span. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Text Types and Purposes*

1. Write arguments to support claims in an analysis of substantive topics or texts using valid reasoning and relevant and sufficient evidence.
2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences.

Production and Distribution of Writing

4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

Research to Build and Present Knowledge

7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

Range of Writing

10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

*These broad types of writing include many subgenres.

Writing Standards for Literacy in History/Social Studies, Science, and Technical Subjects 6–12

The standards below begin at Grade 6; standards for K-5 writing in history/social studies, science, and technical subjects are integrated into the K-5 Writing standards. The CCR anchor standards and high school standards in literacy work in tandem to define college- and career-readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 6-8 Students:	Grades 9-10 Students:	Grades 11-12 Students:
Text Types and Purposes		
1. Write arguments focused on *discipline-specific content*. a. Introduce claim(s) about a topic or issue, acknowledge and distinguish the claim(s) from alternate or opposing claims, and organize the reasons and evidence logically. b. Support claim(s) with logical reasoning and relevant, accurate data and evidence that demonstrate an understanding of the topic or text, using credible sources. c. Use words, phrases, and clauses to create cohesion and clarify the relationships among claim(s), counterclaims, reasons, and evidence. d. Establish and maintain a formal style. e. Provide a concluding statement or section that follows from and supports the argument presented.	1. Write arguments focused on *discipline-specific content*. a. Introduce precise claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that establishes clear relationships among the claim(s), counterclaims, reasons, and evidence. b. Develop claim(s) and counterclaims fairly, supplying data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form and in a manner that anticipates the audience's knowledge level and concerns. c. Use words, phrases, and clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. e. Provide a concluding statement or section that follows from or supports the argument presented.	1. Write arguments focused on *discipline-specific content*. a. Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences the claim(s), counterclaims, reasons, and evidence. b. Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form that anticipates the audience's knowledge level, concerns, values, and possible biases. c. Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. e. Provide a concluding statement or section that follows from or supports the argument presented.

Writing Standards for Literacy in History/Social Studies, Science, and Technical Subjects 6–12 (Continued)

Grades 6-8 Students:	Grades 9-10 Students:	Grades 11-12 Students:
Text Types and Purposes (continued)		
2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style and objective tone. f. Provide a concluding statement or section that follows from and supports the information or explanation presented.	2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. a. Introduce a topic and organize ideas, concepts, and information to make important connections and distinctions; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with well-chosen, relevant, and sufficient facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to manage the complexity of the topic and convey a style appropriate to the discipline and context as well as to the expertise of likely readers. e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).	2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. a. Introduce a topic and organize complex ideas, concepts, and information so that each new element builds on that which precedes it to create a unified whole; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. b. Develop the topic thoroughly by selecting the most significant and relevant facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts. d. Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers. e. Provide a concluding statement or section that follows from and supports the information or explanation provided (e.g., articulating implications or the significance of the topic).
3. (See note; not applicable as a separate requirement)	3. (See note; not applicable as a separate requirement)	3. (See note; not applicable as a separate requirement)

Note: Students' narrative skills continue to grow in these grades. The Standards require that students be able to incorporate narrative elements effectively into arguments and informative/explanatory texts. In history/social studies, students must be able to incorporate narrative accounts into their analyses of individuals or events of historical import. In science and technical subjects, students must be able to write precise enough descriptions of the step-by-step procedures they use in their investigations or technical work so others can replicate them and (possibly) reach the same results.

Writing Standards for Literacy in History/Social Studies, Science, and Technical Subjects 6–12 (Continued)

Grades 6-8 Students:	Grades 9-10 Students:	Grades 11-12 Students:
Production and Distribution of Writing		
4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed.	5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.	5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.
6. Use technology, including the Internet, to produce and publish writing and present the relationships between information and ideas clearly and efficiently.	6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.	6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.
Research to Build and Present Knowledge		
7. Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.	7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.	7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, using search terms effectively; assess the credibility and accuracy of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and following a standard format for citation.	8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the usefulness of each source in answering the research question; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and following a standard format for citation.	8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the specific task, purpose, and audience; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation.
9. Draw evidence from informational texts to support analysis, reflection, and research.	9. Draw evidence from informational texts to support analysis, reflection, and research.	9. Draw evidence from informational texts to support analysis, reflection, and research.
Range of Writing		
10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for range of discipline-specific tasks, purposes, and audiences.

ALABAMA HIGH SCHOOL GRADUATION REQUIREMENTS

(Alabama Administrative Code 290-3-1-02(8) and (8)(a))

Effective for students in the ninth grade in the 2013-2014 school year, all students shall earn the required credits for the Alabama High School Diploma. A local board of education may establish requirements for receipt of diplomas and endorsements, but any diploma or endorsement shall include the requirements of the Alabama High School Diploma. The Alabama courses of study shall be followed in determining minimum required content in each discipline.

COURSE REQUIREMENTS

English Language Arts	**Four credits to include:**	**Credits**
	English 9	**1**
	English 10	**1**
	English 11	**1**
	English 12	**1**
	English Language Arts-credit eligible options may include: Advanced Placement/International Baccalaureate/postsecondary courses/SDE-approved courses.	
	English Language Arts Total Credits	**4**
Mathematics	**Three credits to include:**	**Credits**
	Algebra I or its equivalent/substitute	**1**
	Geometry or its equivalent/substitute	**1**
	Algebra II w/Trigonometry or Algebra II, or its equivalent/substitute	**1**
	Mathematics-credit eligible options may include: *Career and Technical Education*/Advanced Placement/International Baccalaureate/postsecondary courses/SDE-approved courses.	
	One credit from:	
	Alabama Course of Study: Mathematics or mathematics-credit eligible courses from Career and Technical Education/Advanced Placement/International Baccalaureate/postsecondary courses/SDE-approved courses.	**1**
	Mathematics Total Credits	**4**
Science	**Two credits to include:**	**Credits**
	Biology	**1**
	A physical science (Chemistry, Physics, Physical Science)	**1**
	Science-credit eligible options may include: Advanced Placement/International Baccalaureate/postsecondary courses/SDE-approved courses.	
	Two credits from:	
	Alabama Course of Study: Science or science-credit eligible courses from Career and Technical Education/Advanced Placement/International Baccalaureate/postsecondary courses/SDE-approved courses.	**2**
	Science Total Credits	**4**
Social Studies*	**Four credits to include:**	**Credits**
	World History	**1**
	United States History I	**1**
	United States History II	**1**
	United States Government	**0.5**
	Economics	**0.5**
	Social Studies-credit eligible options may include: Advanced Placement/International Baccalaureate/postsecondary courses/SDE-approved courses.	
	Social Studies Total Credits	**4**
Physical Education	Lifelong Individualized Fitness Education (LIFE) or one JROTC Credit	**1**
Health Education		**0.5**
Career Preparedness		**1**
Career and Technical Education (CTE) and/or Foreign Language and/or Arts Education		**3**
Electives		**2.5**
	Total Credits	**24**

GUIDELINES AND SUGGESTIONS FOR LOCAL TIME REQUIREMENTS AND HOMEWORK

Total Instructional Time

The total instructional time of each school day in all schools and at all grade levels shall be not less than 6 hours or 360 minutes, exclusive of lunch periods, recess, or time used for changing classes (*Code of Alabama*, 1975, §16-1-1).

Suggested Time Allotments for Grades 1 - 6

The allocations below are based on considerations of a balanced educational program for Grades 1-6. Local school systems are encouraged to develop a general plan for scheduling that supports interdisciplinary instruction. Remedial and/or enrichment activities should be a part of the time schedule for the specific subject area.

Subject Area	Grades 1-3	Grades 4-6
Language Arts	150 minutes daily	120 minutes daily
Mathematics	60 minutes daily	60 minutes daily
Science	30 minutes daily	45 minutes daily
Social Studies	30 minutes daily	45 minutes daily
Physical Education	30 minutes daily*	30 minutes daily*
Health	60 minutes weekly	60 minutes weekly
Technology Education (Computer Applications)	60 minutes weekly	60 minutes weekly
Character Education	10 minutes daily**	10 minutes daily**
Arts Education		

Dance Music Theatre Visual Arts	*Daily instruction with certified arts specialists in each of the arts disciplines is the most desirable schedule. However, schools unable to provide daily arts instruction in each discipline are encouraged to schedule in Grades 1 through 3 two 30- to 45-minute arts instruction sessions per week and in Grades 4 through 6 a minimum of 60 minutes of instruction per week. Interdisciplinary instruction within the regular classroom setting is encouraged as an alternative approach for scheduling time for arts instruction when certified arts specialists are not available.*

* Established by the Alabama State Department of Education in accordance with *Code of Alabama*, 1975, §16-40-1

** Established by the Alabama State Department of Education in accordance with *Code of Alabama*, 1975, §16-6B-2(h)

Kindergarten

In accordance with *Alabama Administrative Code* r. 290-5-1-.01(5) Minimum Standards for Organizing Kindergarten Programs in Alabama Schools, the daily time schedule of the kindergartens shall be the same as the schedule of the elementary schools in the systems of which they are a part since kindergartens in Alabama operate as full-day programs. There are no established time guidelines for individual subject areas for the kindergarten classroom. The emphasis is on large blocks of time that allow children the opportunity to explore all areas of the curriculum in an unhurried manner.

It is suggested that the full-day kindergarten program be organized utilizing large blocks of time for large groups, small groups, center time, lunch, outdoor activities, snacks, transitions, routines, and afternoon review. Individual exploration, small-group interest activities, interaction with peers and teachers, manipulation of concrete materials, and involvement in many other real-world experiences are needed to provide a balance in the kindergarten classroom.

Grades 7-12

One credit may be granted in Grades 9-12 for required or elective courses consisting of a minimum of 140 instructional hours or in which students demonstrate mastery of Alabama course of study content standards in one credit courses without specified instructional time (*Alabama Administrative Code* r. 290-3-1-.02 (9)(a)).

In those schools where Grades 7 and 8 are housed with other elementary grades, the school may choose the time requirements listed for Grades 4-6 or those listed for Grades 7-12.

Character Education

For all grades, not less than 10 minutes instruction per day shall focus upon the students' development of the following character traits: courage, patriotism, citizenship, honesty, fairness, respect for others, kindness, cooperation, self-respect, self-control, courtesy, compassion, tolerance, diligence, generosity, punctuality, cleanliness, cheerfulness, school pride, respect of the environment, patience, creativity, sportsmanship, loyalty, and perseverance.

Homework

Homework is an important component of every student's instructional program. Students, teachers, and parents should have a clear understanding of the objectives to be accomplished through homework and the role it plays in meeting curriculum requirements. Homework reflects practices that have been taught in the classroom and provides reinforcement and remediation for students. It should be student-managed, and the amount should be age-appropriate, encouraging learning through problem solving and practice.

At every grade level, homework should be meaning-centered and mirror classroom activities and experiences. Independent and collaborative projects that foster creativity, problem-solving abilities, and student responsibility are appropriate. Parental support and supervision reinforce the quality of practice or product as well as skill development.

Each local board of education shall establish a policy on homework consistent with the Alabama State Board of Education resolution adopted February 23, 1984 (Action Item #F-2).

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GLOSSARY

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another.
Example: $\frac{3}{4}$ and $-\frac{3}{4}$ are additive inverses of one another because $\frac{3}{4} + (-\frac{3}{4}) = (-\frac{3}{4}) + \frac{3}{4} = 0$.

Associative property of addition. See Appendix A, Table 3.

Associative property of multiplication. See Appendix A, Table 3.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.

Commutative property. See Appendix A, Table 3.

Complex fraction. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: *computation strategy.*

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: *computation algorithm*.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying “eight,” following this with “nine, ten, eleven. There are eleven books now.”

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Dot plot. See: *line plot.*

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6. (Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method.) See also: *median, third quartile, interquartile range.*

Fraction. A number expressible in the form $\frac{a}{b}$ where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a nonnegative number.) See also: *rational number*.

Identity property of 0. See Appendix A, Table 3.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or –*a* for some whole number *a*.

Interquartile range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. See also: *first quartile, third quartile.*

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line; also known as a dot plot.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Mean or arithmetic mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: If a population grows from 50 to 55 in a year, it grows by $\frac{5}{50}$ = 10% per year.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also: *uniform probability model*.

Properties of equality. See Appendix A, Table 4.

Properties of inequality. See Appendix A, Table 5.

Properties of operations. See Appendix A, Table 3.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a nonzero denominator.

Rational number. A number expressible in the form $\frac{a}{b}$ or $-\frac{a}{b}$ for some fraction $\frac{a}{b}$. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Repeating decimal. The decimal form of a rational number. See also: *terminating decimal*.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships; also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. See also: *median, first quartile, interquartile range*.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. See also: *probability model.*

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ….

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